Part II

Practice Examinations

Prelude

Now that you have learned all the syllabus material of Exam FAM-L, here are four comprehensive practice exams designed to assess your understanding of the whole exam syllabus and boost your chance of passing the real exam.

What are these practice exams like? These practice exams have the following characteristics:

- Each exam has 20 multiple-choice questions distributed in line with the weights of the five topics in Exam FAM-L according to the following table:

Topic		Weight range	Approximate no. of questions (average shown in parentheses)	
7.	Insurance Coverages and Retirement Financial Security Programs	2.5-7.5%	1-3	(2)
8.	Mortality Models	7.5-12.5%	3-5	(4)
9.	Parametric and Non-Parametric Estimation	2.5-7.5%	1-3	(2)
10.	Present Value Random Variables for Long-Term Insurance Coverages	10-15%	4-6	(5)
11.	Premium and Policy Value Calculation for Long-Term Insurance Coverages	15-20%	6-8	(7)
Total		50%	20	

- For your convenience, the questions in these practice exams are sorted according to the five FAM-L exam topics. That is, Question 1 is set on Topic 7 and Question 20 on Topic 11. This way, you can easily see which topics are your weak spots and identify additional practice questions for those topics. Questions in the real exam will appear in a random order.
- They strike a good balance between standard questions testing topics regularly featured in past exams and harder questions testing more unfamiliar topics that were rarely tested in the past, but may play a more important role in Exam FAM-L nowadays. The amount of calculations required by most questions should be reasonable (not too tedious, not trivial).
- The four exams have more or less the same level of difficulty, so you need not work them out in order. You can start with Practice Exam 3 if you like.

How to use these practice exams? To make the most of these practice exams, here are my recommendations:

- Attempt them when and only when you have completed the core of this study manual. Working on the practice exams when you are not fully ready defeats their purpose.
- Set aside exactly 1 hour 45 minutes and work on each exam in a simulated exam environment detached from distractions. Put away your notes and phone—no Facebook, Instagram, Twitter, or Snapchat. You can only have access to the FAM-L tables, the Prometric standard normal calculator, scratch papers, and your calculator, as if this was a real exam.
- Budget your time wisely. Don't spend a disproportionate amount of time (say, more than 10 minutes) on a single question, no matter how difficult it seems, and don't be afraid to skip questions. For a 105-minute exam with 20 questions, you should spend about 5.25 minutes on each question.
- When you are done, check your answers with the detailed illustrative solutions I provide. If you miss a question, it is important to understand the cause. Is it due to a lack of familiarity with the syllabus material, carelessness, or just bad luck? In quite a number of questions, the wrong answers are distractors that come with some rationale. (I tried to anticipate what mistakes students can make. The SOA will do the same!) Even if you get a question right, it is beneficial to look at my solutions, which may be shorter or neater than yours, and may include some problem-solving remarks.

A NOTE OF ENCOURAGEMENT

The experiences of students who took FAM-L recently (and my own experiences) suggest that these practice exams are likely a bit more difficult than the real exam, so don't feel too frustrated if you find the practice exams hard. It is better to see something difficult when you practice than to be under-prepared and caught off guard on the real exam, right?

On the positive side, if you consistently do well (say, you get **at least 15 out of 20 questions** correct in each exam), you should be on your way to passing Exam FAM-L with ease. Good luck!

Practice Exam 1

1. Determine which of the following statements about convertible term insurance is/are true.

I. It offers the policyholder the option to convert the policy to a whole life annuity at the end of the original term.

II. The premium would be recalculated for the new policy.

III. Conversion typically requires evidence of the policyholder's health status at conversion.

(A) I only (B) II only (C) III only
(D) I, II, and III (E) The correct answer is not given by (A), (B), (C), or (D).

2. Which of the following statements about disability income insurance is not true?

(A) The benefits are often capped at 50–70% of the salary that is being replaced.

(B) The elimination period is the time between the beginning of a period of disability and the beginning of the benefit payments.

(C) If the policyholder can do some work, but not at the full earning capacity established before the period of disability, then they may be eligible for a benefit based on partial disability.

(D) The off period is typically selected by the policyholder from a list offered by the insurer.

(E) Return to work assistance offsets costs associated with returning to work after a period of disability.

3. You are given:

$$\mu_{30+t} = \begin{cases} 0.01, & \text{if } 0 \le t < 10, \\ 0.02, & \text{if } 10 \le t < 20, \\ 0.03, & \text{if } 20 \le t. \end{cases}$$

Calculate the 80th percentile of the future lifetime of (30).

(A) 16 (B) 32 (C) 48
(D) 64 (E) 80

4. Fatman, aged 50, is a really fat man. Fully aware of his poor health conditions, he is determined to adopt a healthy lifestyle and lead a normal life one year from now.

You are given:

(i) The force of mortality of Fatman is

$$\mu^*_{50+t} = \begin{cases} 10\mu_{50+t}, & \text{if } 0 \leq t \leq 1, \\ \mu_{50+t}, & \text{if } t > 1, \end{cases}$$

where μ_{50+t}, $t \geq 0$, is the force of mortality of a life aged 50 whose mortality follows the Standard Ultimate Life Table.

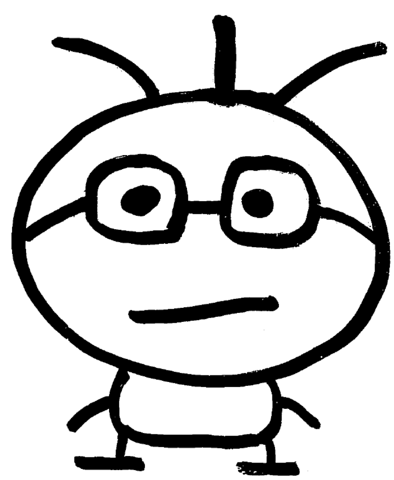
(ii) In the Standard Ultimate Life Table, $e_{50} = 36.09$.

Calculate e^*_{50}, the curtate expectation of life for Fatman.

(**Fatman:** Fat Lives Matter!!!!)

(A) 35.5 (B) 35.6 (C) 35.7
(D) 35.8 (E) 35.9

5. Professor L (shown on the right) only has 3 hairs left on his head and, sadly, he won't be growing any more.



You are given:

(i) The future mortality of each hair follows

$$_{k|}q_x = 0.1(k+1), \quad k = 0, 1, 2, 3$$

and x is Professor L's current age, which is an integer.

(ii) Hair loss is uniformly distributed over each year of age.

(iii) The future lifetimes of the 3 hairs are independent.

Calculate the probability that Professor L is bald (has no hair left) at age $x + 2.5$.

(A) 0.09 (B) 0.11 (C) 0.13
(D) 0.15 (E) 0.17

6. For a 2-year select and ultimate mortality table, you are given:

(i) Ultimate mortality follows the Standard Ultimate Life Table.

(ii) $q_{[x]} = 0.5q_x$ for all x.

(iii) $q_{[x]+1} = 0.7q_{x+1}$ for all x.

Calculate $l_{[90]}$.

(A) 38,000 (B) 39,000 (C) 40,000
(D) 41,000 (E) 42,000

7. You are given the following information regarding deaths for 1000 lives aged 30:

Age at Death	Number of Deaths
30–40	400
40–60	300
60–70	150
70–90	100
90–100	50

Use the ogive to estimate the deferred mortality probability ${}_{25|30}q_{30}$.

(A) 0.1 (B) 0.2 (C) 0.3
(D) 0.4 (E) 0.5

8. For a mortality study of cockroaches, you are given:

(i) 500 cockroaches were observed at time 0.

(ii) An additional 30 cockroaches were first observed at time 2 and y more were first observed at time 5.

(iii) The cage containing the cockroaches was not locked properly, so 23 cockroaches escaped from the cage at time 5. (Watch out! They might be in your room right now...)

(iv) 10 cockroaches died at time 1, 14 died at time 4, 15 died at time 5, and 11 died at time 8.

(v) The Kaplan–Meier estimate of $S(8)$ is 0.9053.

Calculate y.

(A) 10 (B) 20 (C) 30
(D) 40 (E) 50

9. The Actuarial Science Society (ASS)[2] consists of n members all age x today with independent future lifetimes. Having passed Exam FAM all with grade 10, the members apply their expertise in life contingencies to establish a pooled fund to pay a death benefit of \$100 at the end of the year of death for each member. In return, each member contributes a one-time amount of \$50 to this fund at inception.

You are given:

(i) $A_x = 0.455$

(ii) ${}^2A_x = 0.235$

Using the normal approximation without the continuity correction, determine the smallest integer n so that there is at least a 95% probability that the pooled fund will be sufficient to cover the present value of all promised death benefits.

(A) 38 (B) 42 (C) 46
(D) 50 (E) 54

10. For a 2-year term insurance of 1,000 on a smoker aged 65, you are given:

(i) The death benefit is paid at the end of the year of death.

(ii) Mortality of non-smokers follows the Standard Ultimate Life Table.

(iii) The force of mortality of smokers is twice the force of mortality of non-smokers at all ages.

(iv) $i = 0.03$

Calculate the expected present value of this insurance.

(A) 12 (B) 15 (C) 18
(D) 21 (E) 24

11. You are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Deaths are uniformly distributed between integer ages.

(iii) $i = 0.05$

Calculate $1,000\,\mathrm{Var}(\bar{a}_{\overline{T_{55}}|})$.

(A) 8586 (B) 8596 (C) 8606
(D) 8616 (E) 8626

[2] I think a student society with a similar name really exists. See `https://www.melbourneactuary.com/`. ☺

12. In his life contingencies final exam, Guy was asked to calculate

$$\ddot{a}_{x:\overline{n}|}, \qquad a_{x:\overline{n}|}, \qquad \ddot{a}^{(m)}_{x:\overline{n}|}, \qquad a^{(m)}_{x:\overline{n}|},$$

where $m > 1$ is a positive integer, based on the same survival model and positive interest rate. Unprepared for the exam, he cheated and peeked at the answers of his classmate, but only managed to jot the four values in a random order *(bad eyesight!)*:

$$6.373, \qquad 6.539, \qquad 6.128, \qquad 6.789$$

Assuming that the answers of Guy's classmate are correct, calculate m.

(A) 2 (B) 4 (C) 6
(D) 8 (E) 12

13. You are given:

(i) Mortality follows the Standard Ultimate Life Table, which in turn follows the Makeham's law with parameters $A = 0.00022$, $B = 2.7 \times 10^{-6}$, and $c = 1.124$.

(ii) $i = 0.05$

Calculate $1{,}000\,{}_{20|}\ddot{a}^{(12)}_{45}$ using the three-term Woolhouse formula.

(A) 4710 (B) 4711 (C) 4712
(D) 4713 (E) 4714

14. For a fully discrete whole life insurance of 1,000, you are given:

(i) $q_x = 0.10$

(ii) $i = 0.05$

(iii) The annual net premium for this insurance at issue age x is 145.30.

Calculate the annual net premium for this insurance at issue age $x + 1$.

(A) 156 (B) 157 (C) 158
(D) 159 (E) 160

15. For a fully discrete whole life insurance with a face amount of 100,000 issued to (45), you are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Initial expenses are 100 plus 50% of the first year's premium.

(iii) Renewal expenses are 5% of the renewal premiums.

(iv) Claim expenses of 500 are incurred when the death benefit is paid.

(v) $i = 0.05$

(vi) L_0 is the gross loss at issue random variable for this policy.

(vii) The standard deviation of L_0 is 13,095.

Calculate $\mathrm{E}[L_0]$.

(A) $-1,800$ (B) $-1,900$ (C) $-2,000$
(D) $-2,100$ (E) $-2,200$

16. For a fully continuous whole life insurance of 1,000 on (x), you are given:

(i) $\mu_{x+t} = 0.04$ for all $t \geq 0$.

(ii) $\delta = 0.06$

(iii) The premium rate per year is 45.

Calculate the probability that the insurer makes a profit on this policy.

(A) 0.43 (B) 0.47 (C) 0.50
(D) 0.53 (E) 0.57

17. For a 20-year deferred whole life annuity-due of 1,000 per year issued to (45), you are given:

(i) Level annual net premiums of 363 are payable for 20 years.

(ii) $\ddot{a}_{45} = 17.4106$, $\ddot{a}_{55} = 15.7098$, and $\ddot{a}_{65} = 13.4130$.

(iii) ${}_{10}E_{45} = 0.5972$ and ${}_{20}E_{45} = 0.3457$.

Calculate the net premium policy value at the end of year 10.

(A) 4600 (B) 4700 (C) 4800
(D) 4900 (E) 5000

18. For a fully discrete 3-year term insurance of 1,000 on (50), you are given:

(i) $i = 0.06$

(ii) $q_{50} = 0.025$

(iii) The annual net premium is 56.05.

(iv) The net premium policy value at the end of the second year is 29.14.

Calculate q_{51}.

(A) 0.05 (B) 0.06 (C) 0.07
(D) 0.08 (E) 0.09

19. For a fully discrete whole life insurance, you are given:

(i) Level annual premiums, calculated based on the equivalence principle, are paid at the beginning of each year.

(ii) Initial expenses are 50% of the annual gross premium and renewal expenses are 10% of the annual gross premium.

(iii) There are no other expenses.

Determine which of the following statements is/are true.

I. The gross premium is larger than the net premium.

II. The gross premium policy value is larger than the net premium policy value in renewal years.

III. The expense policy value is positive in renewal years.

(A) I only (B) II only (C) III only
(D) I, II, and III (E) The correct answer is not given by (A), (B), (C), or (D).

20. For a fully discrete 20-year endowment insurance of 1,000 on (70), you are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

Calculate the full preliminary term reserve for this insurance at the end of year 10.

(A) 350 (B) 360 (C) 370
(D) 380 (E) 390

****END OF PRACTICE EXAM 1****

Solutions to Practice Exam 1

Answer Key

Question #	Answer	FAM-L Topic
1	B	7
2	D	7
3	D	8
4	C	8
5	A	8
6	A	8
7	C	9
8	D	9
9	A	10
10	E	10

Question #	Answer	FAM-L Topic
11	C	10
12	B	10
13	B	10
14	B	11
15	B	11
16	E	11
17	D	11
18	C	11
19	A	11
20	C	11

Illustrative Solutions

1. [Section 8.2] **(Facts about convertible term insurance)**

Similar example(s)/problem(s): Problem 8.6.3 (which is about renewable term insurance)

Solution. Only II is true. I is false because the option is to convert the term insurance to a whole life insurance, not a whole life annuity. III is also false because conversion does not depend on or require evidence of the policyholder's health status at conversion. **(Answer: (B))** □

2. [Section 8.4] **(Facts about DII)**

Similar example(s)/problem(s): Problems 8.6.15 and 8.6.17

Ambrose's comments: This question tests the specific details of disability income insurance. To complete this question, you need to know the nuts and bolts of DII quite well.

Solution. All statements are true except (D). Unlike the waiting (elimination) period, the off period is typically set by the insurer, not selected by the policyholder from a list offered by the insurer. **(Answer: (D))** □

3. [Section 1.2] **(Percentile of a future lifetime with a piecewise constant μ)**

Similar example(s)/problem(s): Problems 1.1.3 and 1.2.11

Solution. By (1.2.6), the survival function of T_{30} is given by

$$ {}_tp_{30} = \exp\left(-\int_0^t \mu_{30+s}\,\mathrm{d}s\right) = \begin{cases} \mathrm{e}^{-0.01t}, & \text{if } 0 \le t < 10, \\ \mathrm{e}^{-0.01(10)-0.02(t-10)}, & \text{if } 10 \le t < 20, \\ \mathrm{e}^{-0.01(10)-0.02(10)-0.03(t-20)}, & \text{if } 20 \le t. \end{cases} $$

The 80th percentile of T_{30} is the value of t such that ${}_tp_{30} = 0.2$. Since ${}_{20}p_{30} = 0.7408 > 0.2$, the 80th percentile must be larger than 20 and the third expression above applies. Solving

$$ {}_tp_{30} = \mathrm{e}^{-0.01(10)-0.02(10)-0.03(t-20)} = 0.2 $$

yields $t = \boxed{63.65}$ as the required 80th percentile. **(Answer: (D))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(A) This is the 20th percentile of T_{30}, obtained by solving ${}_tp_{30} = 0.8$.

4. [Section 1.5] (e_x with an abnormal first-year mortality)

Similar example(s)/problem(s): Example 1.5.3, and Problems 1.5.10 and 1.5.11

Solution. We can regard the first year ($t = 0$ to $t = 1$) as the period of abnormal mortality, where the SULT force of mortality is multiplied by 10.

- *Standard mortality:* By the one-step recursive formula for e_x,

$$ e_{50} \stackrel{(1.5.3)}{=} p_{50}(1 + e_{51}) = (1 - 0.001209)(1 + e_{51}) = 36.09, $$

  so $e_{51} = 35.1337$.

- *Abnormal mortality:* Since $\mu^*_{50+t} = 10\mu_{50+t}$ for all $t \in [0, 1]$, the one-year survival probability for Fatman is

$$ p^*_{50} \stackrel{(1.2.9)}{=} p^{10}_{50} = (1 - 0.001209)^{10} = 0.987976, $$

  Another application of the one-step recursive formula with $e^*_{51} = e_{51} = 35.1337$ yields

$$ e^*_{50} = p^*_{50}(1 + e^*_{51}) = 0.987976(1 + 35.1337) = \boxed{35.70}. \quad \textbf{(Answer: (C))} $$

□

5. [Section 2.2] (Simple probability calculations with UDD)

Similar example(s)/problem(s): Example 2.2.2

Ambrose's comments: This question about the "mortality" of hair is adapted from an old SOA question (Course 3 Spring 2001 Question 13) testing a fractional age assumption no longer in the exam syllabus.

Solution 1 (Using probability symbols). Using (i) with $k = 0, 1, 2$ and the "$p \times q$" formula for ${}_{u|t}q_x$, we get the mortality rates at ages x, $x + 1$, and $x + 2$:

$$ \begin{cases} {}_{0|}q_x = q_x = 0.1 \\ {}_{1|}q_x \stackrel{(1.1.6)}{=} p_x q_{x+1} = 0.2 \\ {}_{2|}q_x \stackrel{(1.1.6)}{=} p_x p_{x+1} q_{x+2} = 0.3 \end{cases} \quad \Rightarrow \quad \begin{cases} q_x = 0.1 \\ q_{x+1} = 2/9 \\ q_{x+2} = 3/7 \end{cases}. $$

The probability that each hair will "die" before age $x + 2.5$ is

$$ {}_{2.5}q_x = 1 - p_x(p_{x+1})({}_{0.5}p_{x+2}) \stackrel{(2.2.1)}{=} 1 - (1 - 0.1)\left(1 - \frac{2}{9}\right)\left[1 - 0.5\left(\frac{3}{7}\right)\right] = 0.45. $$

As the three hairs have independent future lifetimes, the probability that Professor L is bald at age $x + 2.5$ is $({}_{2.5}q_x)^3 = \boxed{0.0911}$. **(Answer: (A))** □

Solution 2 (Using life table symbols). Assume $l_x = 1$ (any number will do). For $k = 0, 1, 2$,

$$_{k|}q_x \overset{(2.1.4)}{=} \frac{l_{x+k} - l_{x+k+1}}{l_x} = l_{x+k} - l_{x+k+1} \quad \Rightarrow \quad \begin{cases} l_{x+1} = 1 - 0.1 = 0.9 \\ l_{x+2} = 0.9 - 0.2 = 0.7 \\ l_{x+3} = 0.7 - 0.3 = 0.4 \end{cases}.$$

By the UDD assumption, $l_{x+2.5} \overset{(2.2.5)}{=} 0.5(l_{x+2} + l_{x+3}) = 0.55$. Then $_{2.5}q_x = 1 - l_{x+2.5}/l_x = 0.45$. As the three hairs have independent future lifetimes, the probability that Professor L is bald at age $x + 2.5$ is $(_{2.5}q_x)^3 = \boxed{0.0911}$. **(Answer: (A))** □

Remark. The CF assumption would lead to $_{2.5}q_x = 0.470850$.

6. **[Section 2.3] (Going from l_x to $l_{[x]}$)**

Similar example(s)/problem(s): Example 2.3.4 and Problem 2.3.11

Ambrose's comments: This question tests the first type of common exam question about select tables discussed in Subsection 2.3.3 (calculating $l_{[x]}$ in the select table based on the values of l_x in the ultimate table and the relationship between select and ultimate mortality rates).

Solution. With a two-year select period, we can relate $l_{[90]}$ (select, to be determined) and $l_{[90]+2} = l_{92}$ (ultimate, available from SULT) via $l_{[90]}(_2p_{[90]}) = l_{92}$. From (ii) and (iii), the two-year survival probability is

$$\begin{aligned} _2p_{[90]} &= p_{[90]}p_{[90]+1} \\ &= (1 - 0.5q_{90})(1 - 0.7q_{91}) \\ &= [1 - 0.5(0.100917)][1 - 0.7(0.112675)] \\ &= 0.874649. \end{aligned}$$

Then $l_{[90]} = \dfrac{l_{92}}{_2p_{[90]}} = \dfrac{33,379.9}{0.874649} = \boxed{38,163.78}$. **(Answer: (A))** □

Remark. For comparison, $l_{90} = 41,841.1$, corresponding to (E), is larger.

7. **[Section 3.1] (Estimated deferred mortality probability based on ogive)**

Similar example(s)/problem(s): Problems 3.1.1 and 3.1.4

Ambrose's comments: This question tests the material in Section 3.1 about estimation with complete data in a life contingencies context.

Solution. In terms of the survival function for lives aged 30, $_{25|30}q_{30} \overset{(1.1.7)}{=} S_{30}(25) - S_{30}(55)$. We will estimate these two survival probabilities by linear interpolation.

- $\hat{S}_{30}(25)$*:* Time 25 (age 55) lies between time 10 (age 40) and time 30 (age 60). By (3.1.3) and (3.1.4),

$$\begin{aligned} \hat{S}_{30}(25) &= \frac{5\hat{S}_{30}(10) + 15\hat{S}_{30}(30)}{20} \\ &= 0.25\left(1 - \frac{400}{1000}\right) + 0.75\left(1 - \frac{400 + 300}{1000}\right) \\ &= 0.375. \end{aligned}$$

- $\hat{S}_{30}(55)$: Time 55 (age 85) lies between time 40 (age 70) and time 60 (age 90). By (3.1.3) and (3.1.4) again,

$$\begin{aligned}\hat{S}_{30}(55) &= \frac{5\hat{S}_{30}(40)+15\hat{S}_{30}(60)}{20}\\ &= 0.25\left(\frac{100+50}{1000}\right)+0.75\left(\frac{50}{1000}\right)\\ &= 0.075.\end{aligned}$$

The estimated probability is $0.375-0.075=\boxed{0.3}$. **(Answer: (C))** □

8. **[Section 3.2] (Given a Kaplan–Meier estimate, deduce missing information in the data)**

Similar example(s)/problem(s): Example 3.2.3, and Problems 3.2.5, 3.2.6, and 3.2.7

Ambrose's comments: The greatest challenging in this question is perhaps turning the complex description given in the question into a form amenable to calculations. The problems listed above provide useful practice.

Solution. Let's turn the (rather long!) verbal description in the question into the following table showing various quantities at different death times:

$t_{(j)}$	r_j	d_j	c_j	
1	500	10	-30	(30 new entrants at time 2)
4	520	14	0	
5	506	15	$23-y$	(23 exits and y new entrants at time 5)
8	$468+y$	11	–	

Given that $\hat{S}(8)=0.9053$, we solve

$$\left(1-\frac{10}{500}\right)\left(1-\frac{14}{520}\right)\left(1-\frac{15}{506}\right)\left(1-\frac{11}{468+y}\right)=0.9053,$$

so $y=39.7686\approx\boxed{40}$. **(Answer: (D))** □

Remark. Recall that by convention, if there are censoring or truncation times that exactly match a death time (time 5 in this question), then deaths are assumed to occur first, so the 23 cockroaches which escaped at time 5 are part of the risk set at time 5, but the y new entrants are not.

9. **[Section 4.1] (Normal approximation for insurances: How many members should ASS recruit?)**

Similar example(s)/problem(s): Example 4.1.5 and Problem 6.2.51

Ambrose's comments: Instead of testing probabilities or percentiles of a total present value random variable for a given n (size of the group) based on normal approximation, which is regularly featured in past exams, this question turns things around and requires you to calculate the smallest n for a given probability of loss.

Solution. Let $S = \sum_{i=1}^{n} Z_i$, where Z_i is the PVRV of the whole life insurance of \$100 on the ith life, be the total present value of all death benefits. We are interested in the smallest integral value of n such that $\mathrm{P}(S \le 50n) \ge 0.95$, which means that $50n$ should be greater than the 95th percentile of S. From (i) and (ii), we get

$$\mathrm{E}[S] = 100(0.455)n = 45.5n \quad \text{and} \quad \mathrm{Var}(S) = 100^2(0.235 - 0.455^2)n = 279.75n.$$

Using the normal approximation, we solve

$$50n \ge \pi_{0.95}(S) \stackrel{\text{(normal approx.)}}{=} \mathrm{E}[S] + 1.645\sqrt{\mathrm{Var}(S)} = 45.5n + 1.645\sqrt{279.75n},$$

which gives $\sqrt{n} \ge 6.1142$ or $n \ge 37.3832$. In other words, at least $\boxed{38}$ members are needed. **(Answer: (A))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(E) Used 1.96 (97.5th standard normal percentile) in place of 1.645 (95th percentile).

10. [Section 4.1] (EPV of a 2-year discrete term insurance for a smoker)

Similar example(s)/problem(s): Example 4.1.11 and Problem 4.1.42

Solution. As $\mu_x = 2\mu_x^{\text{SULT}}$ for all $x \ge 0$, we have

$$\begin{aligned} p_{65} &\stackrel{(1.2.9)}{=} (p_{65}^{\text{SULT}})^2 = 0.994085^2 = 0.988205, \\ p_{66} &= (p_{66}^{\text{SULT}})^2 = 0.993381^2 = 0.986806. \end{aligned}$$

The EPV of the 2-year term insurance on a smoker is

$$\begin{aligned} 1,000A^{1}_{65:\overline{2}|} &\stackrel{(4.1.2)}{=} 1,000(vq_{65} + v^2 p_{65} q_{66}) \\ &= 1,000\left[\frac{1 - 0.988205}{1.03} + \frac{0.988205(1 - 0.986806)}{1.03^2}\right] \\ &= \boxed{23.74}. \quad \textbf{(Answer: (E))} \end{aligned}$$

□

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(A) This is the value of $1,000A^{1}_{65:\overline{2}|}$ for non-smokers.

(C) This is the average of $1,000A^{1}_{65:\overline{2}|}$ for non-smokers and $1,000A^{1}_{65:\overline{2}|}$ for smokers. Note that you are specifically told to deal with a "2-year term insurance of 1,000 on a *smoker*" and nothing is known about the proportion of smokers and non-smokers in the population.

11. [Sections 4.3 and 5.1] (Variance of a continuous annuity PVRV under UDD)

Similar example(s)/problem(s): Problem 5.1.45

Ambrose's comments: Although this question is not hard, it is educational in the sense that it tests the material in two chapters, Chapters 4 (UDD formulas for insurances) and 5 (variance for annuity PVs).

Solution. By (5.1.10), $\text{Var}(\bar{a}_{\overline{T_{55}}|}) = \dfrac{{}^2\bar{A}_{55} - \bar{A}_{55}^2}{\delta^2}$. Under the UDD assumption, we can calculate the two endowment EPVs as

$$\begin{aligned}\bar{A}_{55} &\overset{(4.3.1)}{=} \frac{i}{\delta}A_{55} = \frac{0.05}{\ln 1.05}(0.23524) = 0.241073,\\ {}^2\bar{A}_{55} &\overset{(4.3.4)}{=} \frac{2i+i^2}{2\delta}({}^2A_{55}) = \frac{2(0.05)+0.05^2}{2\ln 1.05}(0.07483) = 0.078603.\end{aligned}$$

Thus $1,000\,\text{Var}(\bar{a}_{\overline{T_{55}}|}) = 1,000\left[\dfrac{0.078603 - 0.241073^2}{(\ln 1.05)^2}\right] = \boxed{8,606.17}$. **(Answer: (C))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(B) This is $1,000\text{Var}(\ddot{a}_{\overline{K_{55}+1}|}) = 1,000\left[\dfrac{0.07483 - 0.23524^2}{(0.05/1.05)^2}\right] = 8,596.03$.

12. [Section 5.2] (Comparing annuity EPVs, due vs. immediate, annual vs. $1/m$thly)

Similar example(s)/problem(s): Problem 5.2.7

Ambrose's comments: This ambitious and rather challenging question tests both the chain of inequalities in (5.2.7) and the due-immediate formulas relating $a_{x:\overline{n}|}, \ddot{a}_{x:\overline{n}|}$ and $a^{(m)}_{x:\overline{n}|}, \ddot{a}^{(m)}_{x:\overline{n}|}$.

Solution. By (the temporary version of) (5.2.7), $a_{x:\overline{n}|} \le a^{(m)}_{x:\overline{n}|} \le \ddot{a}^{(m)}_{x:\overline{n}|} \le \ddot{a}_{x:\overline{n}|}$, so

$$a_{x:\overline{n}|} = 6.128, \qquad a^{(m)}_{x:\overline{n}|} = 6.373, \qquad \ddot{a}^{(m)}_{x:\overline{n}|} = 6.539, \qquad \ddot{a}_{x:\overline{n}|} = 6.789.$$

Considering $a_{x:\overline{n}|}$ and $\ddot{a}_{x:\overline{n}|}$, we have

$$6.128 = a_{x:\overline{n}|} \overset{(5.1.15)}{=} \ddot{a}_{x:\overline{n}|} - 1 + {}_nE_x = 6.789 - 1 + {}_nE_x,$$

so ${}_nE_x = 0.339$. Then it follows from $a^{(m)}_{x:\overline{n}|}$ and $\ddot{a}^{(m)}_{x:\overline{n}|}$ that

$$6.373 = a^{(m)}_{x:\overline{n}|} \overset{(5.2.5)}{=} \ddot{a}^{(m)}_{x:\overline{n}|} - \frac{1}{m} + \frac{1}{m}{}_nE_x = 6.539 - \frac{1}{m} + \frac{1}{m}(0.339),$$

which gives $m = \boxed{4}$. **(Answer: (B))** □

13. [Section 5.3] (Three-term Woolhouse formula for a deferred annuity)

Similar example(s)/problem(s): Problems 5.3.16 and 5.3.17

Ambrose's comments: Most questions that test the Woolhouse formula center on the two-term formula. This question tests the three-term formula, using the fact that the SULT is based on Makeham's law.

Solution. From page 4 of the FAM-L tables, the force of mortality of the given Makeham's distribution at age 65 is

$$\mu_{65} = A + Bc^{65} = 0.00022 + (2.7\times 10^{-6})(1.124)^{65} = 0.00560485.$$

By the three-term Woolhouse formula,

$$\begin{aligned} {}_{20|}\ddot{a}_{45}^{(12)} &= {}_{20}E_{45}\ddot{a}_{65}^{(12)} \\ &\overset{(5.3.5)}{\approx} {}_{20}E_{45}\left[\ddot{a}_{65} - \frac{12-1}{2(12)} - \frac{12^2-1}{12(12)^2}(\mu_{65}+\delta)\right] \\ &= 0.35994\left[13.5498 - \frac{11}{24} - \frac{143}{1728}(0.00560485 + \ln 1.05)\right] \\ &= 4.710522 \end{aligned}$$

and $1,000\,{}_{20|}\ddot{a}_{45}^{(12)} = \boxed{4,710.52}$. **(Answer: (B))** □

Remark. (i) The Makeham's law is used to get the force of mortality at age 65, which is required for the 3-term Woolhouse formula. If the fact that the SULT is generated from the Makeham's law is not given in the question, then we can apply the approximation

$$\mu_{65} \approx -\frac{1}{2}\ln\left(\frac{l_{66}}{l_{64}}\right) = -\frac{1}{2}\ln\left(\frac{94,020.3}{95,082.5}\right) = 0.005617,$$

which is quite close to the exact value above.

(ii) ⚠ Possible reason(s) for getting some of the incorrect answers:

(A) This is the value based on the UDD approximation:

$${}_{20|}\ddot{a}_{45}^{(12)} \overset{(5.3.1)}{=} {}_{20}E_{45}[\alpha(12)\ddot{a}_{65} - \beta(12)] = 0.35994[1.0002(13.5498) - 0.46651] = 4.7102.$$

(C) This is the value based on the two-term Woolhouse formula:

$${}_{20|}\ddot{a}_{45}^{(12)} \overset{(5.3.6)}{\approx} {}_{20}E_{45}\left[\ddot{a}_{65} - \frac{12-1}{2(12)}\right] = 0.35994\left(13.5498 - \frac{11}{24}\right) = 4.7121.$$

14. [Section 6.1] (Going from P_x to P_{x+1})

Similar example(s)/problem(s): Example 6.1.6 and Problems 6.1.21 and 6.1.25

Ambrose's comments: There is no simple recursive relationship between annual net premiums at consecutive ages, but the net premiums for standard whole life and endowment insurances can be expressed solely in terms of insurance or annuity EPVs, which do enjoy recursive formulas. This observation is the key to solving this problem efficiently.

Solution. By (6.1.1) and (iii),

$$1000P_x = 1000\left(\frac{1}{\ddot{a}_x} - d\right) = 1000\left(\frac{1}{\ddot{a}_x} - \frac{0.05}{1.05}\right) = 145.30 \quad \Rightarrow \quad \ddot{a}_x = 5.183521.$$

Then using the recursive formula for $\ddot{a}_x$, we get

$$5.183521 = \ddot{a}_x \overset{(5.1.6)}{=} 1 + vp_x\ddot{a}_{x+1} = 1 + \frac{1-0.1}{1.05}\ddot{a}_{x+1} \quad \Rightarrow \quad \ddot{a}_{x+1} = 4.880775$$

One more application of (6.1.1), this time at age $x+1$, yields the required net premium at age $x+1$:

$$1000P_{x+1} = 1000\left(\frac{1}{\ddot{a}_{x+1}} - \frac{0.05}{1.05}\right) = 1000\left(\frac{1}{4.880775} - \frac{0.05}{1.05}\right) = \boxed{157.27}.$$ **(Answer: (B))**

□

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(C) Mistakenly used $P_x = \dfrac{1}{\ddot{a}_x} - i$ instead of $P_x = \dfrac{1}{\ddot{a}_x} - d$.

15. **[Section 6.2] (From variance to expected value of L_0)**

Similar example(s)/problem(s): Example 6.2.10, and Problems 6.2.26 and 6.2.31

Solution. We can deduce the gross premium from the standard deviation in (vii). By (6.2.1) with $S = 100,000$, $E = 500$, and $G - R = 0.95G$,

$$\sqrt{\mathrm{Var}(L_0^g)} = \sqrt{\left(100,500 + \frac{0.95G}{0.05/1.05}\right)^2 (0.03463 - 0.15161^2)} = 13,095,$$

so $G = 1,045.2111$. Then the expected value of the gross loss at issue random variable is

$$\begin{aligned} \mathrm{E}[L_0^g] &= 100,500A_{45} + (100 + 0.45G) - 0.95G\ddot{a}_{45} \\ &= 100,500(0.15161) + [100 + 0.45(1,045.2111)] - 0.95(1,045.2111)(17.8162) \\ &= \boxed{-1,883.46}. \quad \textbf{(Answer: (B))} \end{aligned}$$

□

16. **[Section 6.2] (Probability of a profit for a fully continuous whole life insurance)**

Similar example(s)/problem(s): Example 6.2.15, and Problems 6.2.44 and 6.2.45

Ambrose's comments: Remember that besides EPV and variance, other actuarial quantities commonly tested in the exam include probabilities and percentiles of PVRVs (of insurances, annuities, or PV of losses). This question tests a probability for a loss at issue random variable.

Solution. In terms of T_x, the loss at issue random variable is

$$L_0 = 1000v^{T_x} - 45\bar{a}_{\overline{T_x}|} = 1000v^{T_x} - 45\left(\frac{1 - v^{T_x}}{0.06}\right) = 1750v^{T_x} - 750.$$

Therefore,

$$L_0 < 0 \quad \Leftrightarrow \quad v^{T_x} < \frac{3}{7} \quad \Leftrightarrow \quad \mathrm{e}^{-0.06T_x} < \frac{3}{7} \quad \Leftrightarrow \quad T_x > -\frac{\ln(3/7)}{0.06} = 14.1216,$$

and the required probability is $\mathrm{P}(T_x > 14.1216) = \mathrm{e}^{-0.04(14.1216)} = \boxed{0.5684}$. **(Answer: (E))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(A) This is the probability of a loss, $1 - 0.5684 = 0.4316$.

17. **[Section 7.1] (Policy value of a deferred annuity by definition)**

Ambrose's comments: Calculating the policy value of a deferred annuity from first principles is rarely tested in past exams. This question serves to fill this void.

Solution. By definition, the net premium policy value at the end of year 10 is ${}_{10}V = 1000({}_{10|}\ddot{a}_{55}) - P\ddot{a}_{55:\overline{10|}}$. The two annuity factors can be calculated as

$$\begin{aligned} {}_{10|}\ddot{a}_{55} &\overset{(5.1.18)}{=} {}_{10}E_{55}\ddot{a}_{65} \overset{(4.1.3)}{=} \frac{0.3457}{0.5972}(13.4130) = 7.764357, \\ \ddot{a}_{55:\overline{10|}} &\overset{(5.1.17)}{=} \ddot{a}_{55} - {}_{10|}\ddot{a}_{55} = 15.7098 - 7.764357 = 7.945443, \end{aligned}$$

we have ${}_{10}V = 1000(7.764357) - 363(7.945443) = \boxed{4880.16}$. **(Answer: (D))** □

Remark. The annual net premium given in (i) can be calculated as

$$P = \frac{1000({}_{20|}\ddot{a}_{45})}{\ddot{a}_{45:\overline{20|}}} = \frac{1000({}_{20}E_{45}\ddot{a}_{65})}{\ddot{a}_{45} - {}_{20}E_{45}\ddot{a}_{65}} = \frac{1000(0.34570)(13.4130)}{17.4106 - 0.34570(13.4130)} = 363.0009.$$

This question would probably be too cumbersome if you had to calculate both the net premium and the net premium policy value.

18. [Section 7.2] (Two applications of the policy value recursive formula)

Similar example(s)/problem(s): Problems 7.2.20 and 7.2.21

Solution. Applying the policy value recursive formula at $t = 0$ and noting that ${}_0V^n = 0$, we have

$$(0 + 56.05)(1.06) = 0.025(1,000) + (1 - 0.025)({}_1V^n),$$

so ${}_1V^n = 35.295385$. One more application of the recursive formula, in the NAAR form and this time at $t = 1$, yields

$$(35.295385 + 56.05)(1.06) = 29.14 + (1,000 - 29.14)q_{51},$$

which gives $q_{51} = \boxed{0.0697}$. **(Answer: (C))** □

Remark. (i) The fact that the insurance is "3-year term" plays no role in this question.

(ii) ⚠ Possible reason(s) for getting some of the incorrect answers:

(E) This is the value of q_{52}, which is not what the question requires, but can be obtained by applying the recursive formula at $t = 2$:

$$(29.14 + 56.05)(1.06) = 0 + (1,000 - 0)q_{52} \quad \Rightarrow \quad q_{52} = 0.0903.$$

19. [Section 7.2] (Gross vs. net premiums, and gross vs. net premium policy values)

Similar example(s)/problem(s): Problem 7.2.56

Solution. I. True. The gross premium is always larger than the net premium because the former has to fund the expenses as well. Mathematically, $P^g = P^n + P^e > P^n$ because $P^e > 0$.

II-III. False. With front-loaded expenses (which is the case in this question, according to (ii)), the expense loading is larger than the renewal expenses, leading to a negative expense policy value in renewal years, i.e., ${}_tV^e < 0$ for all $t > 0$.

As a result, the gross premium policy value is smaller than the net premium policy value in renewal years. Mathematically, ${}_tV^g = {}_tV^n + {}_tV^e < {}_tV^n$ for all $t > 0$. **(Answer: (A))** □

20. **[Section 7.2] (FPT reserve for an endowment insurance)**

Similar example(s)/problem(s): Example 7.2.23 and Problem 7.2.74

Solution. By (7.2.14), ${}_{10}V^{\text{FPT}} = 1,000\left(1 - \dfrac{\ddot{a}_{80:\overline{10}|}}{\ddot{a}_{71:\overline{19}|}}\right)$. From the SULT, $\ddot{a}_{80:\overline{10}|} = 6.7885$ and $\ddot{a}_{70:\overline{20}|} = 11.1109$. By the recursive formula for $\ddot{a}_{x:\overline{n}|}$,

$$\ddot{a}_{70:\overline{20}|} \overset{(5.1.14)}{=} 1 + vp_{70}\ddot{a}_{71:\overline{19}|} \quad \Rightarrow \quad \ddot{a}_{71:\overline{19}|} = \frac{11.1109 - 1}{(1 - 0.010413)/1.05} = 10.728157.$$

Hence ${}_{10}V^{\text{FPT}} = 1,000\left(1 - \dfrac{6.7885}{10.728157}\right) = \boxed{367.23}$. **(Answer: (C))** □

Remark. (i) You can also calculate $\ddot{a}_{71:\overline{19}|}$ as $\ddot{a}_{71} - {}_{19}E_{71}\ddot{a}_{90} = \ddot{a}_{71} - \dfrac{1}{1.05^{19}}\left(\dfrac{l_{90}}{l_{71}}\right)\ddot{a}_{90}$, but the calculations are slightly more involved than using the recursive formula for $\ddot{a}_{x:\overline{n}|}$.

(ii) ⚠ Possible reason(s) for getting some of the incorrect answers:

(E) This is the net premium policy value ${}_{10}V^n = 1,000\left(1 - \dfrac{\ddot{a}_{80:\overline{10}|}}{\ddot{a}_{70:\overline{20}|}}\right) = 389.02$.

